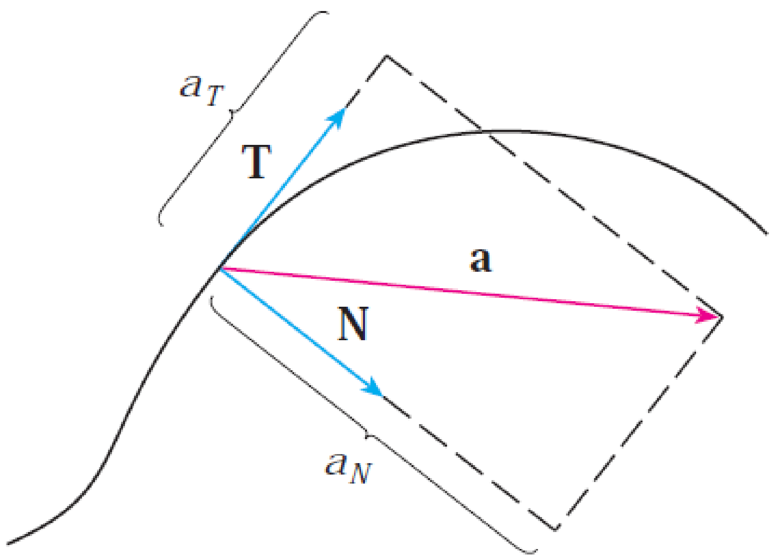
Closing Thu: 13.4, 14.1

13.4 (Cont.) Measuring acceleration



Entry Task:

Given $\boldsymbol{r}(t) =< x(t), y(t), z(t) >$

Write down the definition/formula for:

1. the velocity
2. the speed
3. the acceleration
4. the curvature
5. the unit tangent
6. the unit normal

Recall: $\text{comp}_{\boldsymbol{b}}(\boldsymbol{a}) = \frac{\boldsymbol{a}\cdot\boldsymbol{b}}{\boldsymbol{b}}$ = length.

Thus, the tangential and normal components of acceleration are:

$a_T = \text{comp}_{\boldsymbol{T}}(\boldsymbol{a}) = \boldsymbol{a} \cdot \boldsymbol{T}$ = tangential

$a_N = \text{comp}_{\boldsymbol{N}}(\boldsymbol{a}) = \boldsymbol{a} \cdot \boldsymbol{N}$ = normal

Note that: $\boldsymbol{a} = a_T\boldsymbol{T} + a_N\boldsymbol{N}$

For computing:

$$a_T = \frac{\vec{\boldsymbol{r}}' \cdot \vec{\boldsymbol{r}}''}{|\vec{\boldsymbol{r}}'|} \quad \text{and} \quad a_T = \frac{|\vec{\boldsymbol{r}}' \times \vec{\boldsymbol{r}}''|}{|\vec{\boldsymbol{r}}'|}$$

For interpretation:

$a_T = \nu' = \frac{d}{dt}|r'(t)| =$ "deriv. of speed"

$a_N = k\nu^2 =$ curvature · (speed)^2

Deriving an interpretation:

Let $\nu(t) = |\vec{\boldsymbol{v}}(t)|$ = speed.

1. $\vec{\boldsymbol{T}}(t) = \frac{\vec{\boldsymbol{r}}'(t)}{|\vec{\boldsymbol{r}}'(t)|} = \frac{\vec{\boldsymbol{v}}(t)}{\nu(t)}$ implies $\vec{\boldsymbol{v}} = \nu\vec{\boldsymbol{T}}$.
2. $\kappa(t) = \frac{|\vec{\boldsymbol{T}}'(t)|}{|\vec{\boldsymbol{r}}'(t)|} = \frac{|\vec{\boldsymbol{T}}'|}{\nu(t)}$ implies $|\vec{\boldsymbol{T}}'| = \kappa\nu$.
3. $\vec{\boldsymbol{N}}(t) = \frac{\vec{\boldsymbol{T}}'(t)}{|\vec{\boldsymbol{T}}'(t)|} = \frac{\vec{\boldsymbol{T}}'}{\kappa\nu}$,implies $\vec{\boldsymbol{T}}' = \kappa\nu\vec{\boldsymbol{N}}$.

Differentiating the first fact above gives

$\vec{\boldsymbol{a}} = \vec{\boldsymbol{v}}' = \nu'\vec{\boldsymbol{T}} + \nu\vec{\boldsymbol{T}}'$, so

$\vec{\boldsymbol{a}} = \vec{\boldsymbol{v}}' = \nu'\vec{\boldsymbol{T}} + k\nu^2\vec{\boldsymbol{N}}$.

Conclusion

$a_T = \nu' = \frac{d}{dt}|\boldsymbol{r}'(t)| =$ "deriv. of speed"

$a_N = k\nu^2 =$ curvature · (speed)^2

Example:

$$\vec{\boldsymbol{r}}(t) = < \cos(t)\,, \sin(t)\,, t >$$

Find the tangential and normal components of acceleration.

14.1/14.3 Intro to Multivariable Functions and Partial Derivatives

Def'n: A function, *f*, of two variables is a rule that assigns a number for each input (x,y).
We write

$$z = f(x,y).$$

We graph the function in 3D: (x,y) is the location on the xy-plane and z is the height above that point.

We sometimes write $f: \mathbb{R}^2 \to \mathbb{R}$.

The set of allowable inputs is called the **domain**. The domain will be a region in 2-dimensions. The set of possible outputs is called the **range**.

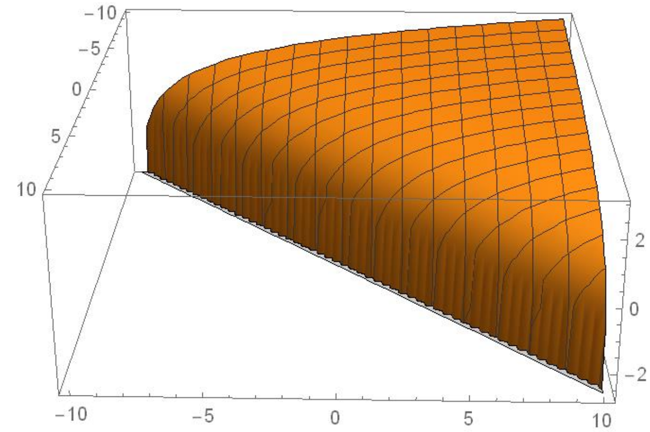
Domain: Any question that asks "find the domain" is simply asking you if you know your functions well enough to understand when they are not defined.

Appears in Function	*Restriction*
$\sqrt{BLAH}$	BLAH ≥ 0
STUFF/BLAH	BLAH ≠ 0
ln(BLAH)	BLAH > 0
$\sin^{-1}$(BLAH)	-1 ≤ BLAH ≤ 1
and other trig…	

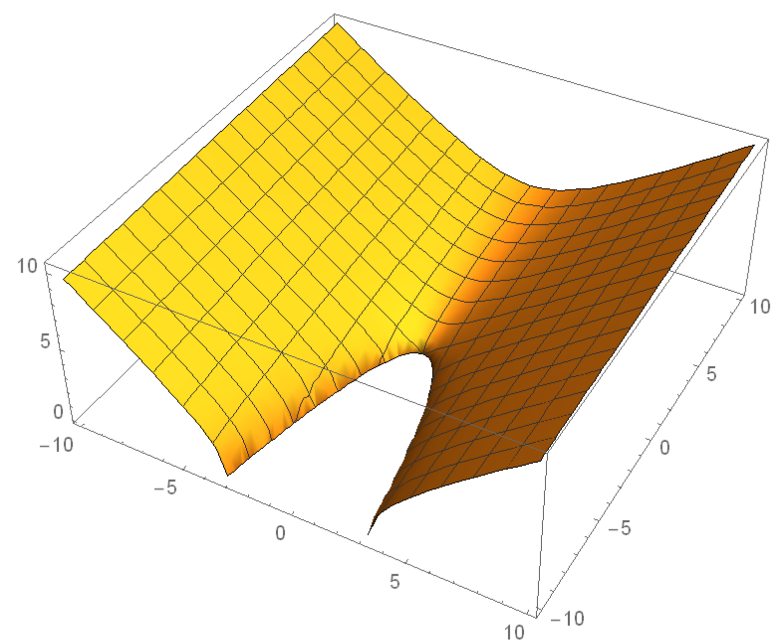
Examples:

Sketch the domain of

(1) $f(x, y) = \ln(y - x)$



(2) $g(x, y) = \sqrt{y + x^2}$



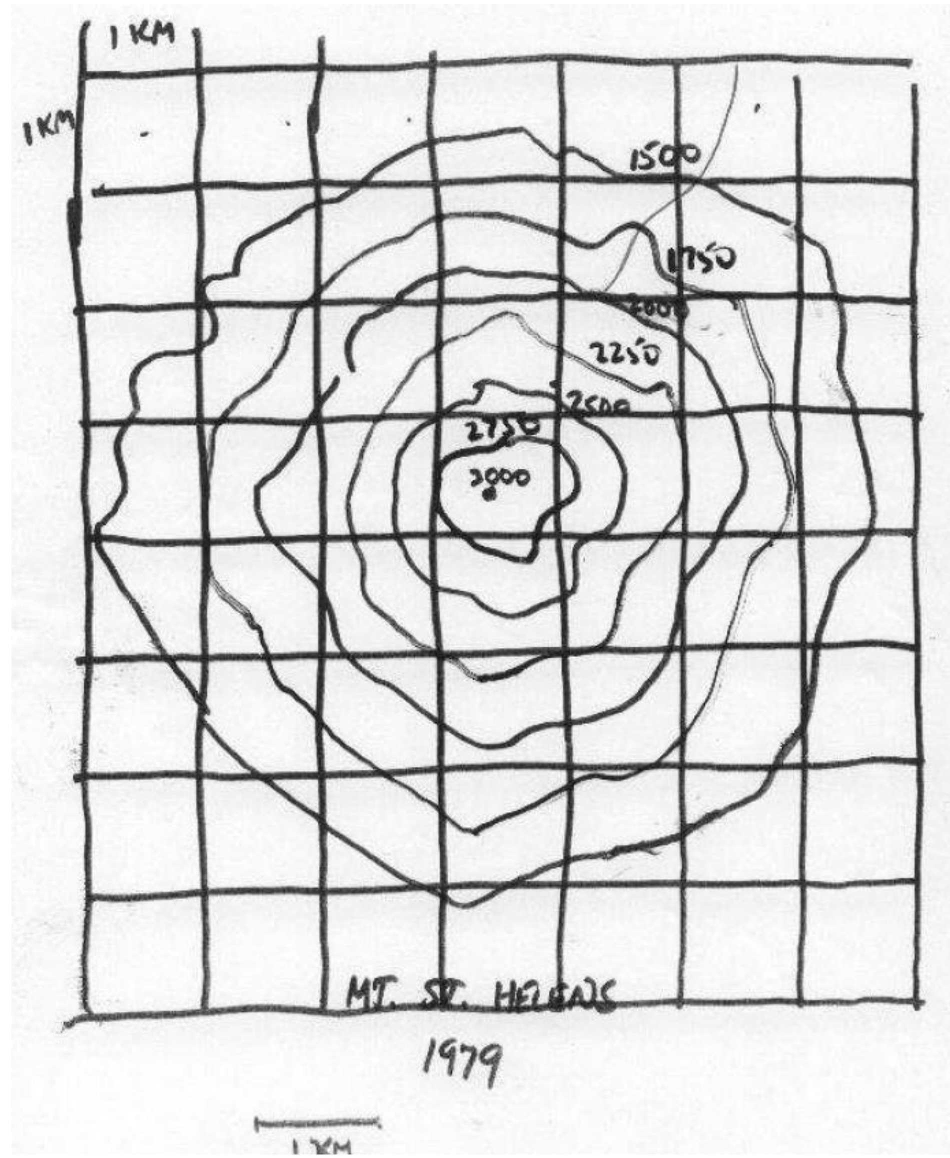
Visualizing Surfaces

The basic tool for graphing surfaces is **traces**. When $z = f(x, y)$, we typically look at traces given by fixed values of z (height) first.

We call these traces **level curves**, because each curve represents all the points at the same height (level) on the surface.

A collection of level curves is called a **contour map** (or **elevation map**).

Contour Map (Elevation Map) of Mt. St. Helens from 1979 (before it erupted):

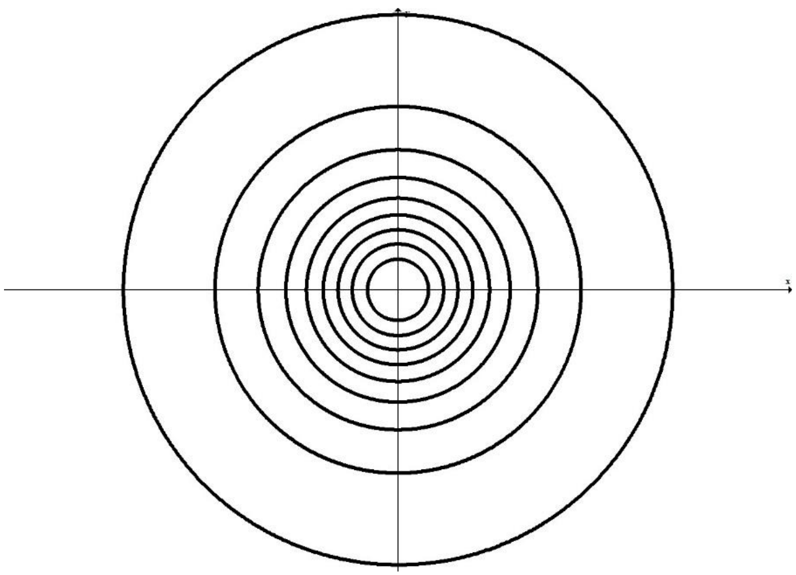


Examples:

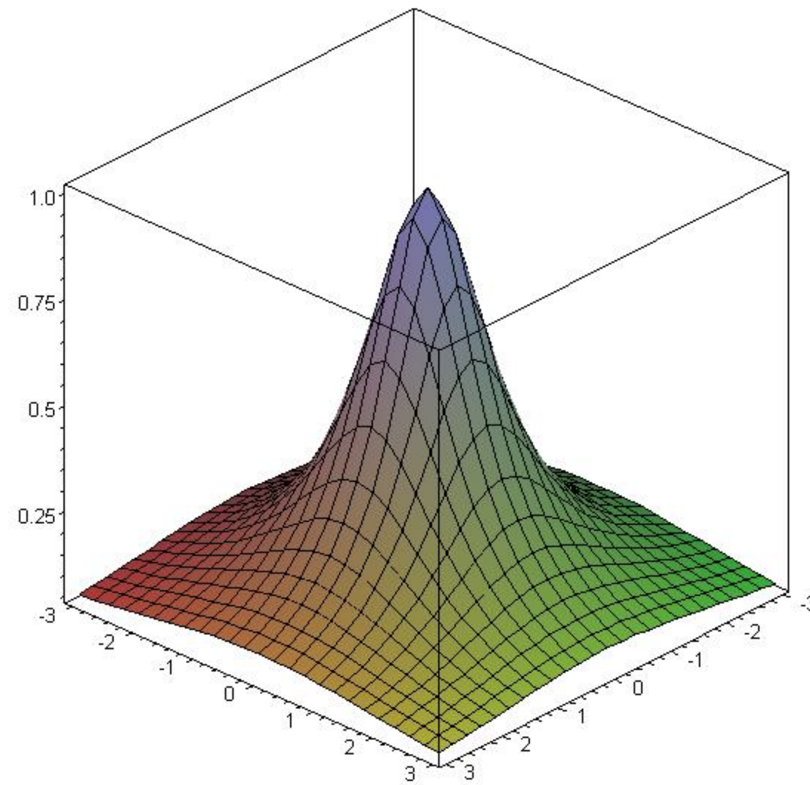
Level Curves for

$$z = f(x,y) = \frac{1}{1+x^2+y^2}$$

at z = 1/10, 2/10, … , 9/10, 10/10



Graph of $z = f(x,y) = \frac{1}{1+x^2+y^2}$



Graph of $z = \sin(x) - y$